Optimal and reliable design of timber trusses considering decay degradation in aggressive environment

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1. Context of the study
2. Modeling timber decay
3. Design optimization approaches
4. Numerical application
5. Conclusion

Context of the study

- This work is a part of the results of CLIMBOIS project financed by the French National Agency of Research.
- The mechanical and physical properties of timber structures are **affected** by a combination of loading, moisture content, temperature, biological activity, etc.
- Humid exposure and larger wood moisture content **induce** high risk for bio deterioration of unprotected timber (mould and fungal decay).

Decay: reduces the strength of timber and is the main factor to be considered in assessing the durability of wooden structures.

Context of the study

Resource rationalization

Safety Assurance

Uncertainty of material parameters

Model analysis approximations

Optimization

Decision making tools

Material degradation (decay, aging,…)

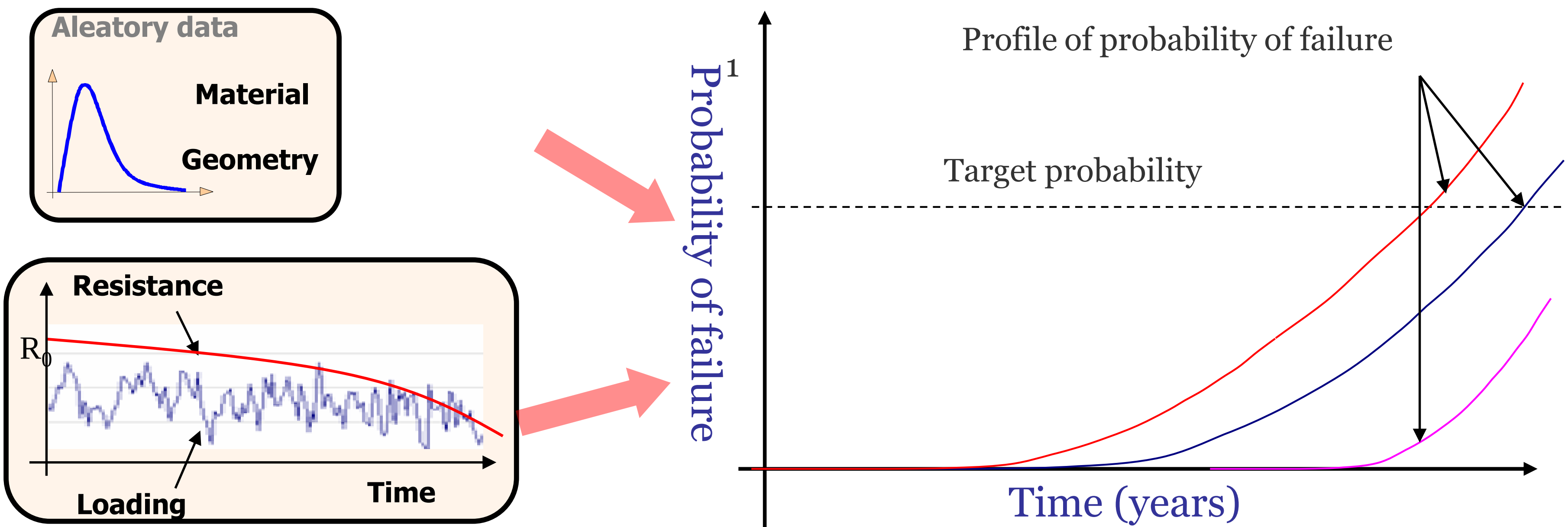
Loading and climate fluctuations

Reliability-Based Design Optimization offers a suitable framework for the consideration of the uncertainties in the design optimization and to find the best compromise between cost reduction and safety assurance.

Objectives of the study

➢ **How to optimize the structural design of timber structures subjected to decay and climate variations with considering uncertainties?**

- Uncertainty from material properties, geometrical dimensions, numerical models.
- Imperfect knowledge of the structural and material degradation.
- Fluctuation of loads and weather actions (temperature, humidity, etc.)



Modeling timber decay

- Viitanen et al [Viitanen 2010] → model for the decay growth of brown rot in pine sapwood under variant climate conditions.
- The model is divided into two processes: (i) activation process and (ii) mass loss process.

➢ Activation process

- A parameter α is used as a relative measure of fungi deterioration activity.
- α is set initially to 0. Once it reaches the limit value α=1, the mass loss initiates

$$\alpha(t) = \sum_{i=0}^{t} \Delta\alpha(i) \text{ with } \alpha(t) \in [0,1]$$

where

$$\Delta\alpha(i) = \begin{cases} \dfrac{\Delta t}{t_{crit}(RH(i),T(i))} & \text{if } T(i) > 0°\text{C and } RH(i) > 95\% \\ -\dfrac{\Delta t}{17520} & \text{otherwise} \end{cases}$$

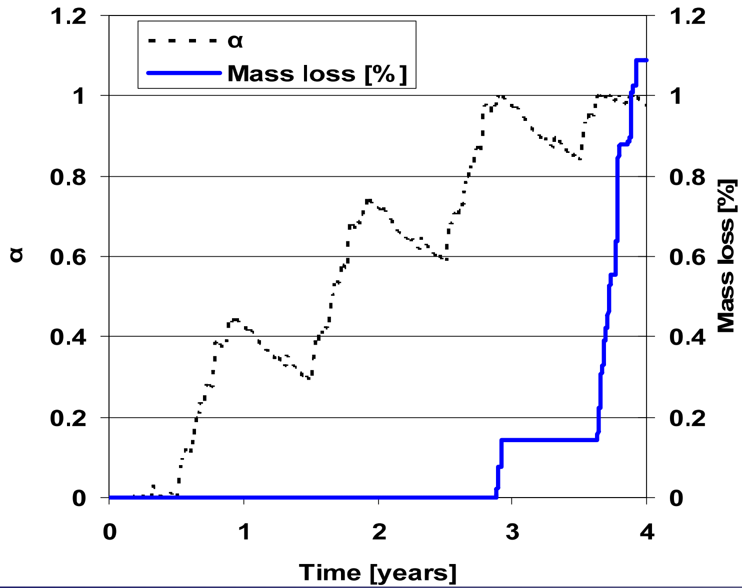
Modeling timber decay

➢ Mass loss process

- Mass loss (in % of initial weight) occurs once the fungi activation process is reached, ($\alpha(t) = 1$) and it is estimated as:

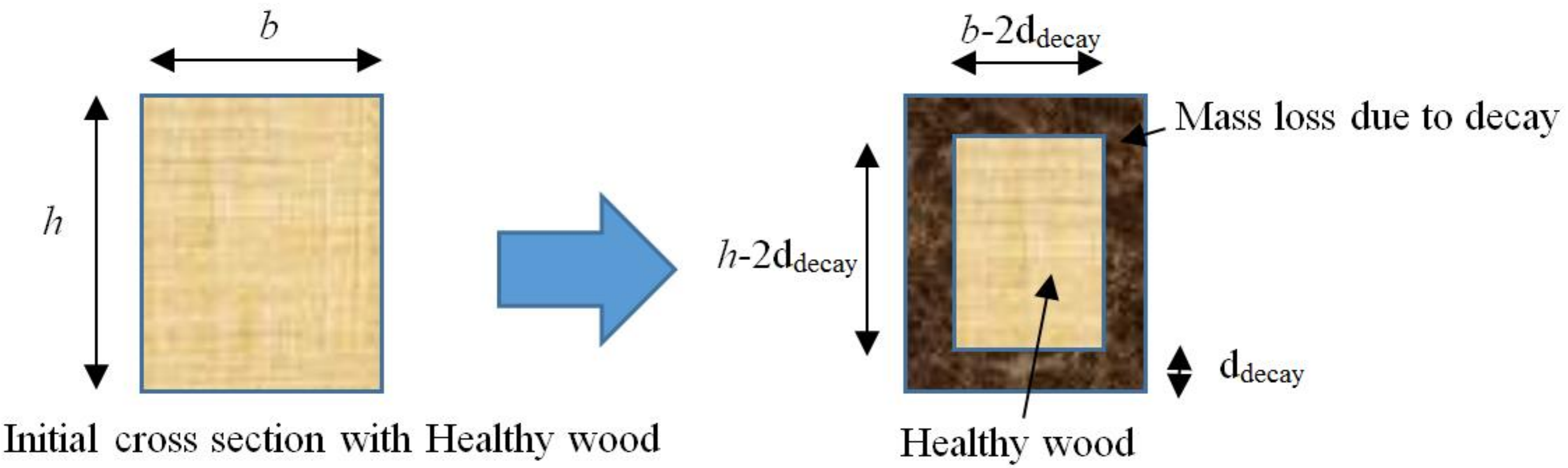
$$ML(t) = \sum_{i=0}^{t} \left(\frac{ML(RH(i), T(i))}{dt} \times \Delta t \times 1_{\alpha}(i) \right)$$

- Mass loss only takes place when the temperature is above 0°C and the relative humidity is above 95%. Otherwise the mass loss process is stopped.



Loss of mechanical performance

- The depth of decay attack is estimated on the basis of the mass loss.

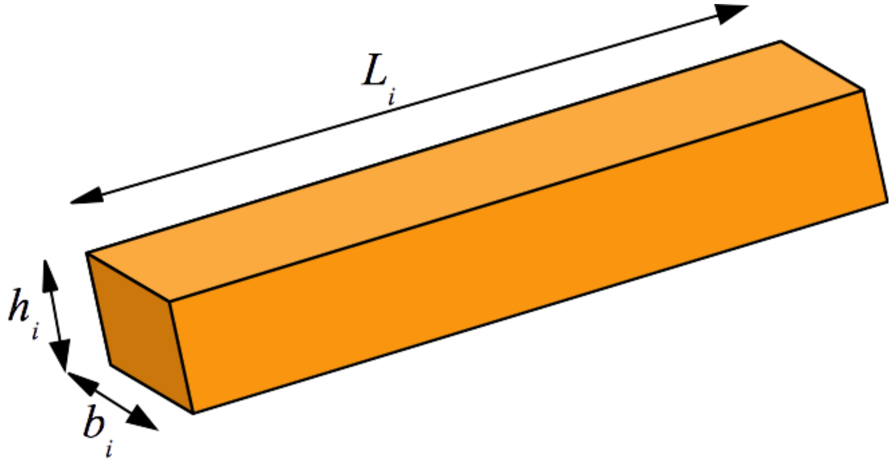


Deterministic optimization approach of timber trusses

- Searching the optimal design **d** that minimizes an objective function defined in terms of structural cost or volume:

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho L_i h_i b_i$$

$$\text{subject to} \begin{cases} G_{ULS,i}\left(\mathbf{d},\mathbf{x}_k,\gamma\right) \leq 0 \\ G_{SLS,i}\left(\mathbf{d},\mathbf{x}_k,\gamma\right) \leq 0 \\ \psi_i\left(\mathbf{d}\right) \leq 0 \end{cases}$$



- **d**: vector of design variables (depth b_i and height h_i of the i^th member).
- L_i: length of the i^{th} member and ρ is the timber density.
- $G_{ULS,i}$ and $G_{SLS,i}$ are respectively the i^{th} Ultimate Limit State (ULS) and Service Limit State (SLS) that representing stress and deflection constraints.
- $\mathbf{x}_k$: vector containing the characteristic values of load actions and material properties
- γ: partial safety factors and ψ are the feasibility constraints (e.g. upper and lower bounds of design variables).

Insufficiency of deterministic optimization approach

Application of the partial safety factors

- The safety margins are not linked to the target reliability.
- The partial safety factors are calibrated for a large class of structures.

Over design

Unreliable design

Rational approach

- Uncertainty quantification.
- Probabilistic modeling of the safety margin.

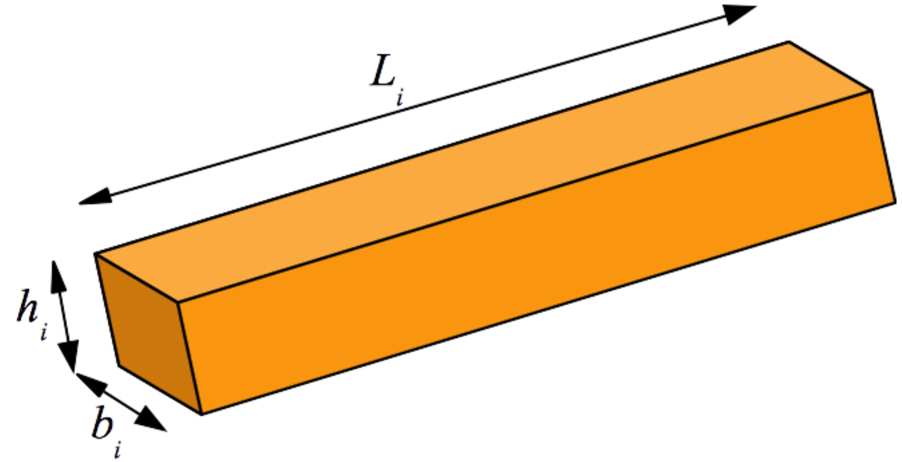
Reliability-based design optimization (RBDO) of timber trusses

- Searching the optimal design **d** that minimizes an objective function defined in terms of structural cost or volume:

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho\, L_i h_i b_i$$

subject to

$$\begin{cases} Prob\left[G_{ULS_i}(\mathbf{d},\mathbf{X}) \leq 0\right] \leq P_{f_i}^{t_{ULS}} \\ Prob\left[G_{SLS_i}(\mathbf{d},\mathbf{X}) \leq 0\right] \leq P_{f_i}^{t_{SLS}} \\ \psi_i(\mathbf{d}) \leq 0 \end{cases}$$



$P_{f_i}^{t_{ULS}}$: target probability of failure for ultimate limit state G_{ULS}

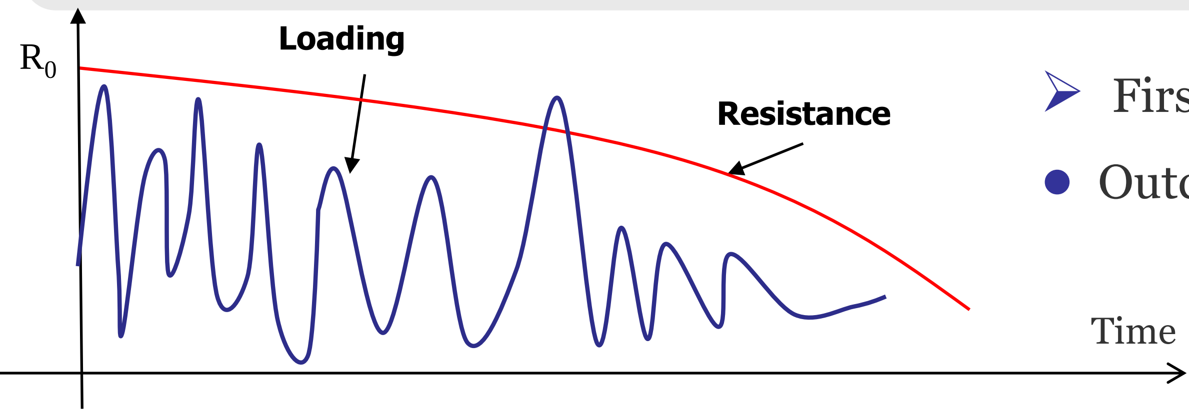
$P_{f_i}^{t_{SLS}}$: target probability of failure for serviceability limit state G_{SLS}

ψ: feasibility constraints (e.g. upper and lower bounds of design variables).

Time-variant reliability

Time-variant reliability

- Consider material degradation and load fluctuations with time (stochastic process)



- First passage of the limit state
- Outcrossing rate approach.

$$\nu(t) = \lim_{\Delta t \to 0} \frac{P\left[\left(G\left(t, X(t,\omega)\right) \in D_s\right) \bigcap \left(G\left(t+\Delta t, X(t+\Delta t,\omega)\right) \in D_f\right)\right]}{\Delta t}$$

- The cumulative probability of failure is approximated with:

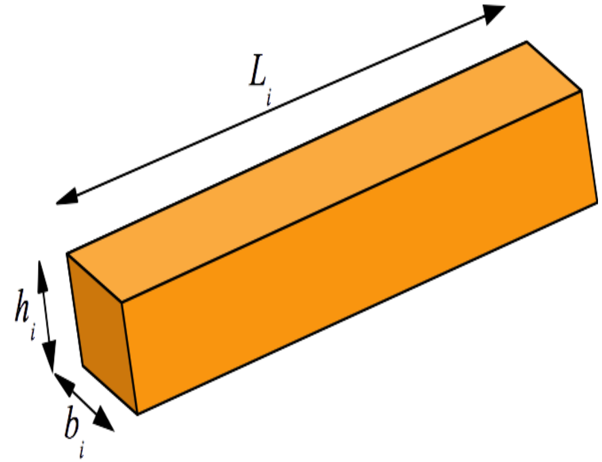
$$P_{f,c}\left(0,T\right) \approx P_{f,i}(0) + \int_0^T \nu\left(t\right)\, dt$$

Time-variant RBDO

$$\min_{\mathbf{d}=\{\mathbf{h},\mathbf{b}\}} \sum_{i=1}^{n_b} \rho L_i h_i b_i$$

subject to

$$\begin{cases} \forall t \in [0, T_L],\ Prob\left[G_{ULS_i}(\mathbf{d}, \mathbf{X}, t) \le 0\right] \le P_f^{T_L, ULS} \\ \forall t \in [0, T_L],\ Prob\left[G_{SLS_i}(\mathbf{d}, \mathbf{X}, t) \le 0\right] \le P_f^{T_L, SLS} \\ \psi_i(\mathbf{d}) \le 0 \end{cases}$$

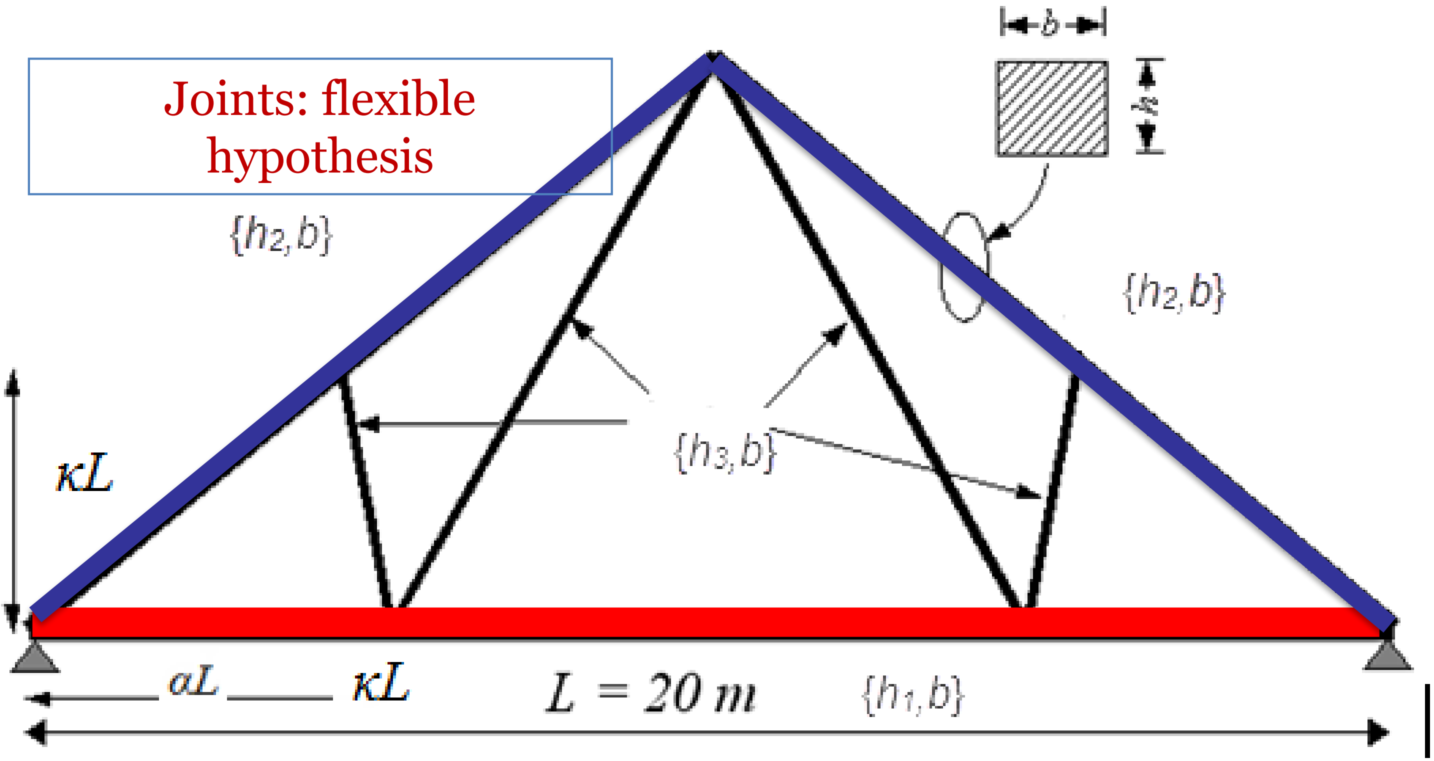


$P_f^{T_L, ULS}$: is the target probability of failure for ultimate limit state G_{ULS} at the allowable life time T_L

$P_f^{T_L, SLS}$: is the target probability of failure for serviceability limit state G_{SLS} at the allowable life time T_L

Timber roof optimal designs

- Compare the optimized designs obtained from the **DDO** (Deterministic Design Optimization) and **TV-RBDO** (Time-Variant Reliability-Based Design Optimization) methods for a timber roof truss subjected to decay.
- Estimate and compare the time-variant reliability profiles of each optimized solution.



Limit state functions

- Ultimate Limit State (ULS):
  - tension and bending: where the tension is parallel to the grain;
  - compression: where members are checked for compressive strength as well as for buckling;
  - shear: for all the truss members for a timber roof truss subjected to decay.
  - The target reliability index for one year is fixed to 4.7 [NF-EN 1990]

- Serviceability Limit State (SLS):
  - instantaneous deflection;
  - final deflection composed with the instantaneous and creep deflections.
  - The target reliability index for one year is fixed to 2.9 [NF-EN 1990]

Target reliability index

Table C2 - Target reliability index β for Class RC2 structural members [1)]

Limit state	Target reliability index	
	1 year	50 years
Ultimate	4,7	3,8
Fatigue		1,5 to 3,8 [2)]
Serviceability (irreversible)	2,9	1,5

[1)] See Annex B
[2)] Depends on degree of inspectability, reparability and damage tolerance.

- The target reliability index at the allowable life time T_L, depending on the target reliability related to one year reference period by:

$$\beta_{T_L}^{c} = \Phi^{-1}\left(\Phi\left(\beta_1^{t}\right)^{T_L}\right)$$

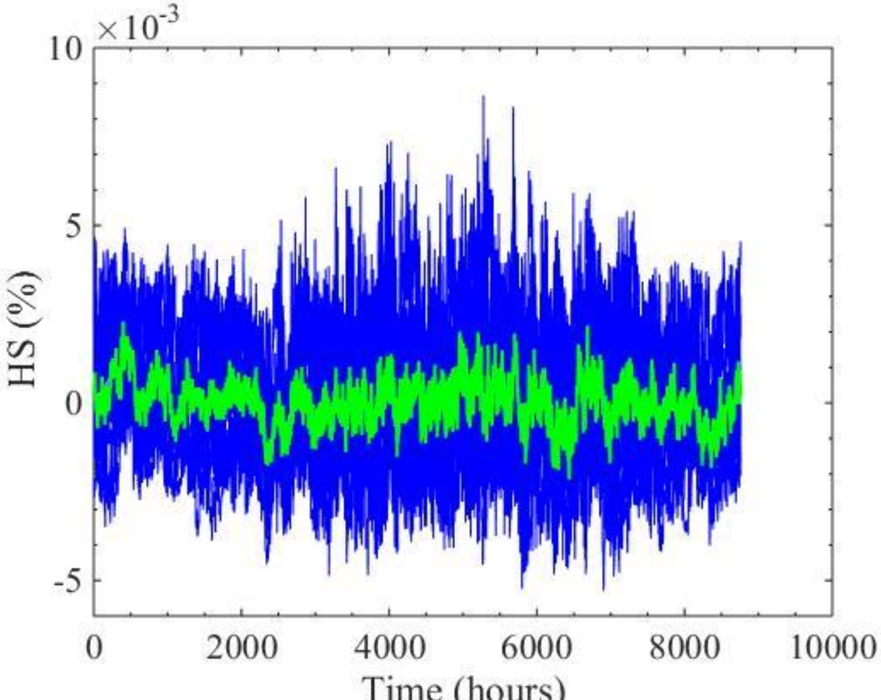
- The target reliability index for 30 years for ULS is 3.95
- The target reliability index for 30 years for SLS is 1.60

Statistical parameters for materials and loads

Name parameter	Characteristic value	Mean value	Coefficient of variation
f_m (MPa)	24	37.1	0.25
f_c (MPa)	21	29.7	0.20
$f_{c,90}$ (MPa)	2.5	3.5	0.20
f_t (MPa)	14	23.7	0.30
f_v (MPa)	4	5.65	0.20
E (MPa)	10908	11000	0.13
Dead load (kN/m²)	620	466.5	0.10
Snow (kN/m²)	1193	798.8	0.3
Wind (kN/m²)	1320	883.9	0.3

Probabilistic model of the relative humidity

- Exposure to a very humid environment corresponding to the city of **Nantes** in France during 30 years.
- Nantes: annual mean temperature of **12.7 ºC** and relative humidity of **81%.**



Simulated relative humidity values during one year

Optimal designs of the timber roof

Name parameter	DDO optimal design	TV-RBDO optimal design
b (mm)	220	240
h_1 (mm)	294	320
h_2 (mm)	342	481
h_3 (mm)	293	320
Truss weight (kg)	1812	2418
ULS Reliability index at 30 years (target : 3.95)	**2.54**	**3.95**
SLS Reliability index at 30 years (target :1.60)	**1.58**	**4.39**

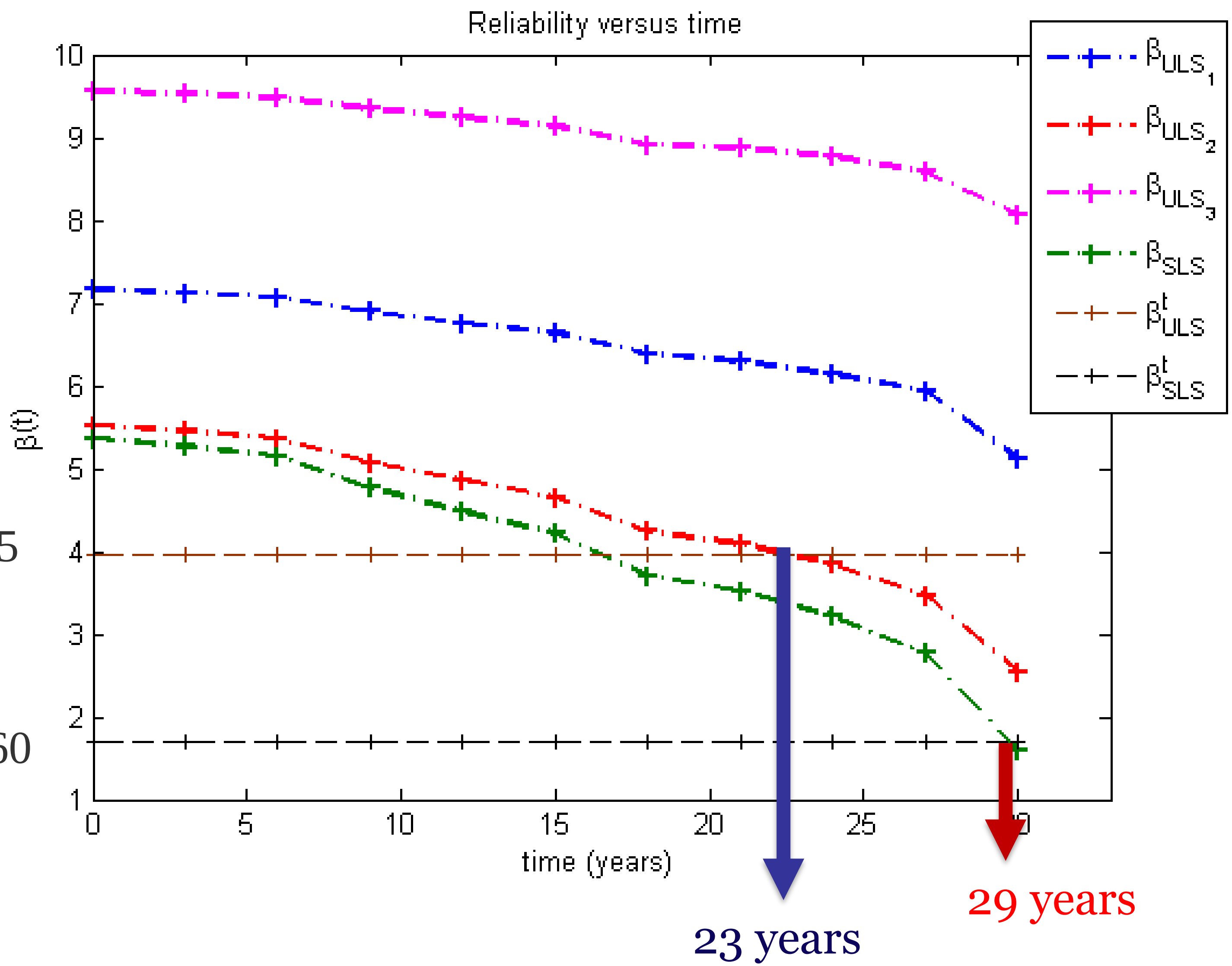
Optimal designs of the timber roof

- The optimal design of the truss obtained by TV-RBDO is more reliable than that obtained by the deterministic approach.
- The optimal design of the TV-RBDO satisfies the reliability lifetime requirement.
- The TV-RBDO design solution is more expensive, where the optimal weight is 1.33 times more than the DDO weight.

Time-variant reliability of the DDO design solution

$\beta^{t}_{SLS,30} = 3.95$

$\beta^{t}_{SLS,30} = 1.60$



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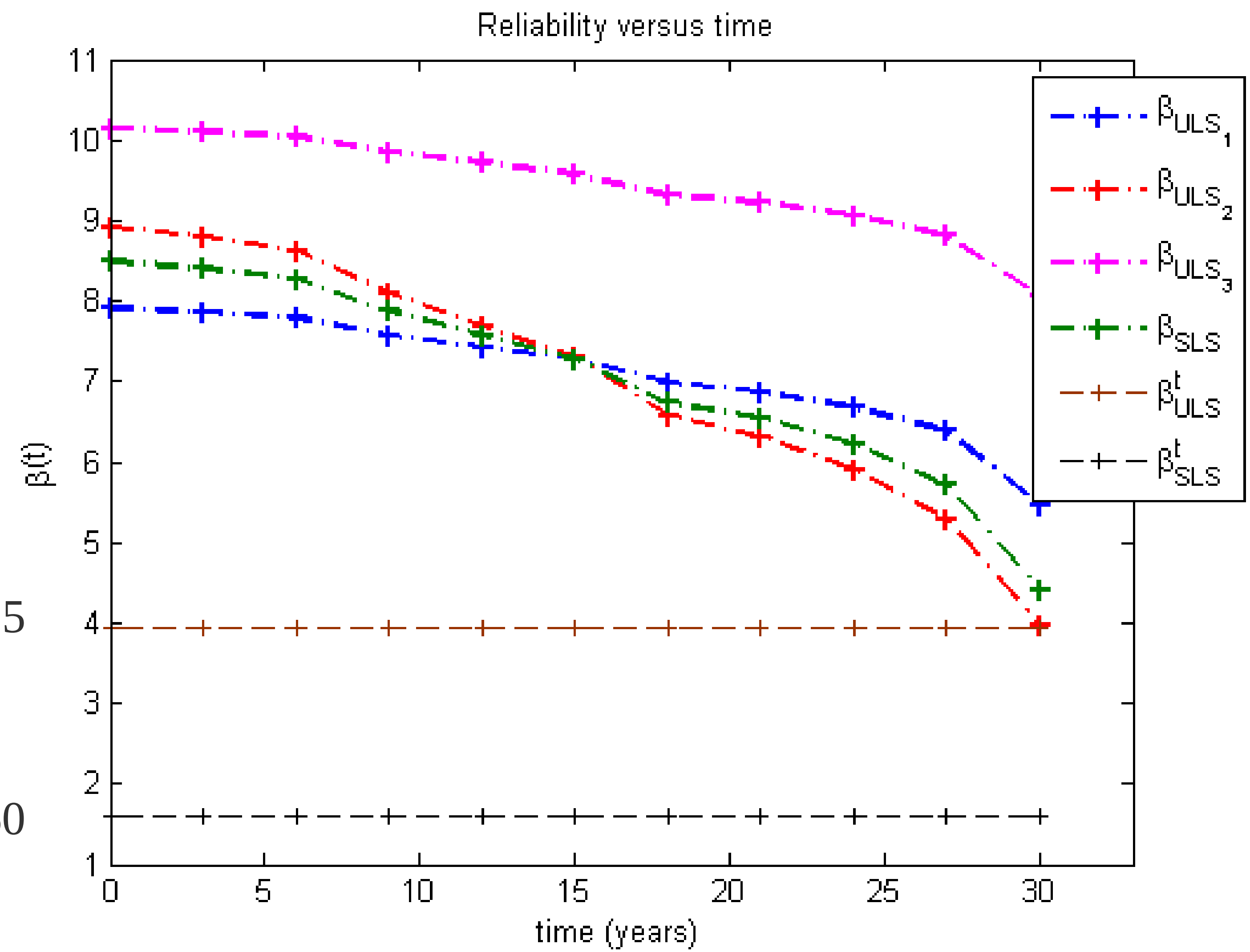
Time-variant reliability of the TV-RBDO design solution

$\beta^{t}{}_{SLS,30} = 3.95$

$\beta^{t}{}_{SLS,30} = 1.60$



Conclusion

- Design optimization of timber structures by accounting for uncertainties, climate variations, deterioration, ultimate and serviceability constraints.

- The use of the partial safety factors in the deterministic design optimization cannot guarantee the target reliability, these partial safety factors are not calibrated to take into account decay .

- The use of the TV-RBDO to search the optimal design that minimizes the structural cost and ensures the target reliability level during the operational life.

- Optimal calibration of the partial safety factors in the TV-RBDO approach allow us to ensure the best compromise between cost reduction and safety assurance.

- The potential benefits of the TV-RBDO approach to design roof trusses considering decay.

The authors would like to express their appreciation to the French National Agency of Research (ANR) for its financial support of this work through the project CLIMBOIS Project No. ANR-13-JS09-0003-01 as well as the labeling of the ViaMéca French cluster. The authors thank also the COST Action FP1303 for the financial support to attend this final conference.

Thank you for your kind attention!